

ENHANCED GLUINO BOX CONTRIBUTION TO ϵ'/ϵ IN SUSY

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We show that in supersymmetric extensions of the Standard Model gluino box diagrams can yield a large CP-violating $\Delta I = \frac{3}{2}$ contribution to $s \rightarrow d\bar{q}q$ flavor-changing neutral current processes, which feeds into the $I = 2$ isospin amplitude in $K \rightarrow \pi\pi$ decays. This contribution only requires moderate mass splitting between the right-handed squarks \tilde{u}_R and \tilde{d}_R , and persists for squark masses of order 1 TeV. Taking into account current bounds on $\text{Im } \delta_{sd}^{LL}$ from $K-\bar{K}$ mixing, the resulting contribution to ϵ'/ϵ could naturally be an order of magnitude larger than the measured value.

The recent confirmation of direct CP violation in $K \rightarrow \pi\pi$ decays is an important step in testing the Cabibbo–Kobayashi–Maskawa (CKM) mechanism for CP violation in the Standard Model. Combining the recent measurements by the KTeV and NA48 experiments with earlier results from NA31 and E731 gives $\text{Re } (\epsilon'/\epsilon) = (2.12 \pm 0.46) \times 10^{-3}$. This value tends to be higher than theoretical predictions in the Standard Model, which center below or around 1×10^{-3} . Unfortunately, it is difficult to gauge the accuracy of these predictions, because they depend on hadronic matrix elements which at present cannot be computed from first principles. A Standard-Model explanation of ϵ'/ϵ can therefore not be excluded. Nevertheless, it is interesting to ask how large ϵ'/ϵ could be in extensions of the Standard Model.

In the context of supersymmetric models, it has been known for some time that it is possible to obtain a large contribution to ϵ'/ϵ via the $\Delta I = \frac{1}{2}$ chromomagnetic dipole operator without violating constraints from $K-\bar{K}$ mixing.¹ It has recently been pointed out that this mechanism can naturally be realized in various supersymmetric scenarios.² Here we discuss a new mechanism involving a supersymmetric contribution to ϵ'/ϵ induced by $\Delta I = \frac{3}{2}$ penguin operators.³ These operators are potentially important because their effect is enhanced by the $\Delta I = \frac{1}{2}$ selection rule. Unlike recent proposals, which involve left-right down-squark mass insertions, our effect relies on the left-left insertion δ_{sd}^{LL} and requires (moderate) isospin violation in the right-handed squark sector.

The ratio ϵ'/ϵ parameterizing the strength of direct CP violation in $K \rightarrow \pi\pi$ decays can be expressed as

$$\frac{\epsilon'}{\epsilon} \simeq \frac{\omega}{\sqrt{2}|\epsilon|} \left(\frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0} \right), \quad (1)$$

where A_I are the isospin amplitudes for the decays $K^0 \rightarrow (\pi\pi)_I$, and the ratio

$\omega = \text{Re } A_2 / \text{Re } A_0 \approx 0.045$ signals the strong enhancement of $\Delta I = \frac{1}{2}$ transitions over those with $\Delta I = \frac{3}{2}$. In the Standard Model, the amplitudes A_I receive small, CP-violating contributions via the ratio $(V_{ts}^* V_{td}) / (V_{us}^* V_{ud})$ of CKM matrix elements, which enters through the interference of the $s \rightarrow u\bar{u}d$ tree diagram with penguin diagrams involving the exchange of a virtual top quark. According to (1), contributions to ϵ'/ϵ due to the $\Delta I = \frac{3}{2}$ amplitude $\text{Im } A_2$ are enhanced relative to those due to the $\Delta I = \frac{1}{2}$ amplitude $\text{Im } A_0$ by a factor of $\omega^{-1} \approx 22$. However, in the Standard Model the dominant CP-violating contributions to ϵ'/ϵ are due to QCD penguin operators, which only contribute to A_0 . Penguin contributions to A_2 arise through electroweak interactions and are suppressed by a power of α . Their effects on ϵ'/ϵ are small and of the same order as isospin-violating effects such as $\pi^0 - \eta - \eta'$ mixing.

In supersymmetric extensions of the Standard Model potentially large, CP-violating contributions can arise from flavor-changing *strong-interaction* processes induced by gluino box diagrams. Whereas in the limit of exact isospin symmetry in the squark sector these graphs only induce $\Delta I = \frac{1}{2}$ operators at low energies, in the presence of even moderate up-down squark mass splitting they can lead to operators with large $\Delta I = \frac{3}{2}$ components. In the terminology of the standard effective weak Hamiltonian, this implies that the supersymmetric contributions to the Wilson coefficients of QCD and electroweak penguin operators can be of the same order. Specifically, both sets of coefficients scale like α_s^2 / \tilde{m}^2 with \tilde{m} a generic supersymmetric mass, compared with $\alpha_s \alpha_W / m_W^2$ and $\alpha \alpha_W / m_W^2$, respectively, in the Standard Model. These contributions can be much larger than the electroweak penguins of the Standard Model even for supersymmetric masses of order 1 TeV.

The relevant $\Delta S = 1$ gluino box diagrams lead to the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^4 \left[c_i^q(\mu) Q_i^q(\mu) + \tilde{c}_i^q(\mu) \tilde{Q}_i^q(\mu) \right] + \text{h.c.},$$

where

$$\begin{aligned} Q_1^q &= (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V+A}, & Q_2^q &= (\bar{d}_\alpha s_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_3^q &= (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V-A}, & Q_4^q &= (\bar{d}_\alpha s_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A} \end{aligned}$$

are local four-quark operators, \tilde{Q}_i^q are operators of opposite chirality obtained by interchanging $V - A \leftrightarrow V + A$, and a summation over $q = u, d, \dots$ and over color indices α, β is implied. In the calculation of the coefficient functions we use the mass insertion approximation, in which case the gluino-quark-squark couplings are flavor diagonal. Flavor mixing is due to small deviations from squark-mass degeneracy and is parameterized by dimensionless quantities δ_{ij}^{AB} , where i, j are squark flavor indices and A, B refer to the chiralities of the corresponding quarks. In general, these mass insertions can carry new CP-violating phases. Contributions involving left-right squark mixing are neglected, since they are quadratic in small mass insertion parameters, i.e., proportional to $\delta_{sd}^{LR} \delta_{qq}^{LR}$. We define the dimensionless mass ratios $x_u^{L,R} = (m_{\tilde{u}_{L,R}} / m_{\tilde{g}})^2$ and $x_d^{L,R} = (m_{\tilde{d}_{L,R}} / m_{\tilde{g}})^2$, where $m_{\tilde{u}_{L,R}}$ and $m_{\tilde{d}_{L,R}}$ denote the

Table 1: Imaginary parts of the coefficients $\Delta c_{1,2}(m_c)$ in units of $10^{-4} \text{Im} \delta_{sd}^{LL}$, for gluino and down-squark masses of 500 GeV and different values of $m_{\tilde{u}_R}$. The last column shows the corresponding values in the Standard Model.

$m_{\tilde{u}_R}$ [GeV]	750	1000	1500	SM
$\text{Im} \Delta c_1(m_c)$	-0.05	-0.08	-0.12	0.42×10^{-7}
$\text{Im} \Delta c_2(m_c)$	2.12	3.19	4.16	-1.90×10^{-7}

average left- or right-handed squark masses in the up and down sector, respectively. $\text{SU}(2)_L$ gauge symmetry implies that the mass splitting between the left-handed up- and down-squarks must be tiny; however, we will not assume such a degeneracy between the right-handed squarks.

It is straightforward to relate the quantities c_i^q to the Wilson coefficients appearing in the effective weak Hamiltonian of the Standard Model. The supersymmetric contributions to the electroweak penguin coefficients vanish in the limit of universal squark masses. However, for moderate up-down squark mass splitting the differences $\Delta c_i \equiv c_i^u - c_i^d$ are of the same order as the coefficients c_i^q themselves. In this case gluino box contributions to QCD and electroweak penguin operators are of similar magnitude. Because the electroweak penguin operators contain $\Delta I = \frac{3}{2}$ components their contributions to ϵ'/ϵ are strongly enhanced and thus are expected to be an order of magnitude larger than the contributions from the QCD penguin operators. In this talk, we focus only on these enhanced contributions. Since the mass splitting between the left-handed \tilde{u}_L and \tilde{d}_L squarks is tiny, we can safely neglect the coefficients $\Delta c_{3,4}$ and $\Delta \tilde{c}_{1,2}$ in our analysis. Finally, in estimating the supersymmetric contribution to ϵ'/ϵ we focus only on the $(V-A) \otimes (V+A)$ operators associated with the coefficients Δc_1 and Δc_2 , because their matrix elements are chirally enhanced with respect to those of the $(V+A) \otimes (V+A)$ operators.

Table 1 shows the imaginary parts of the coefficients $\Delta c_{1,2}$ obtained for the illustrative choice $\tilde{m} = m_{\tilde{g}} = m_{\tilde{d}_L} = m_{\tilde{d}_R} = 500 \text{ GeV}$ and three (larger) values of $m_{\tilde{u}_R}$. For fixed ratios of the supersymmetric masses the values of the coefficients scale like \tilde{m}^{-2} , i.e., significantly larger values could be obtained for smaller masses. For comparison, the last column contains the imaginary parts of $\Delta c_{1,2}$ in the Standard Model. We observe that for supersymmetric masses of order 500 GeV, and for a mass insertion parameter $\text{Im} \delta_{sd}^{LL}$ of order a few times 10^{-3} (see below), the imaginary part of Δc_2 can be significantly larger than that of the corresponding coefficient in the Standard Model, which is proportional to C_8 . This is interesting, since even in the Standard Model the contribution of C_8 to ϵ'/ϵ is significant.

The penguin operators contribute to the imaginary part of the isospin amplitude A_2 . The real part is, to an excellent approximation, given by the matrix elements of the standard current-current operators in the effective weak Hamiltonian. Evaluating the matrix elements of the four-quark operators in the factorization approximation, and parameterizing nonfactorizable corrections by a hadronic parameter

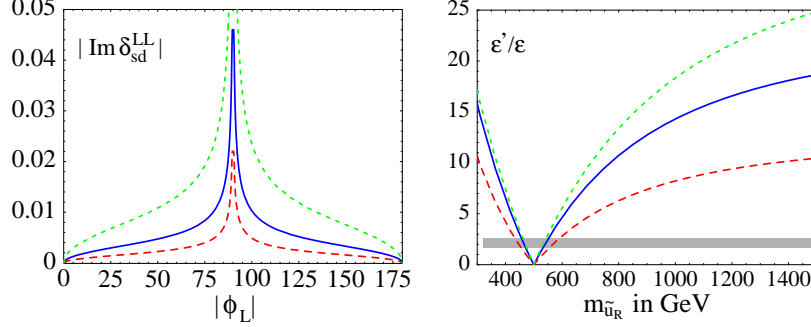


Figure 1: Left: Upper bound on $|\text{Im } \delta_{sd}^{LL}|$ versus the weak phase $|\phi_L|$ (in degrees) for $m_{\tilde{d}_L} = 500$ GeV and $(m_{\tilde{g}}/m_{\tilde{d}_L})^2 = 1$ (solid), 0.3 (dashed) and 4 (short-dashed). Right: Supersymmetric contribution to $|\epsilon'/\epsilon|$ (in units of 10^{-3}) versus $m_{\tilde{u}_R}$, for $m_s(m_c) = 130$ MeV, $B_8^{(2)}(m_c) = 1$, $m_{\tilde{d}_L} = m_{\tilde{d}_R} = 500$ GeV, and $(m_{\tilde{g}}/m_{\tilde{d}_L})^2 = 1$ (solid), 0.3 (dashed) and 4 (short-dashed). The values of $|\text{Im } \delta_{sd}^{LL}|$ corresponding to the three curves are 0.011, 0.005 and 0.027, respectively (see text). The band shows the average experimental value.

$B_8^{(2)}$, we find for the supersymmetric $\Delta I = \frac{3}{2}$ contribution to ϵ'/ϵ

$$\frac{\epsilon'}{\epsilon} \approx 19.2 \left[\frac{500 \text{ GeV}}{m_{\tilde{g}}} \right]^2 \left[\frac{\alpha_s(\tilde{m})}{0.096} \right]^{\frac{34}{21}} \left[\frac{130 \text{ MeV}}{m_s(m_c)} \right]^2 B_8^{(2)}(m_c) X(x_d^L, x_u^R, x_d^R) \text{Im } \delta_{sd}^{LL},$$

where $X(x, y, z)$ is a known function of SUSY mass ratios. The existence of this contribution requires a new CP-violating phase ϕ_L defined by $\text{Im } \delta_{sd}^{LL} \equiv |\delta_{sd}^{LL}| \sin \phi_L$. The measured values of Δm_K and ϵ in $K-\bar{K}$ mixing give bounds on $\text{Re } (\delta_{sd}^{LL})^2$ and $\text{Im } (\delta_{sd}^{LL})^2$, respectively, which can be combined to obtain a bound on $\text{Im } \delta_{sd}^{LL}$ as a function of ϕ_L . Using the most recent analysis of supersymmetric contributions to $K-\bar{K}$ mixing in Ref. ⁴, we show in the left-hand plot in Fig. 1 the results obtained for $m_{\tilde{d}_L} = 500$ GeV and three choices of $m_{\tilde{g}}$. It is evident that the bound on $\text{Im } \delta_{sd}^{LL}$ depends strongly on the precise value of ϕ_L . To address the issue of how large a supersymmetric contribution to ϵ'/ϵ one can reasonably expect from our mechanism, it appears unnatural to take the absolute maximum of the bound given the peaked nature of the curves. We thus evaluate our result taking for $\text{Im } \delta_{sd}^{LL}$ one quarter of the maximal allowed values. In this way, we obtain a plausible upper bound on the SUSY contribution to ϵ'/ϵ , which does not require fine-tuning.

Our results for $|\epsilon'/\epsilon|$ are shown in the right-hand plot in Fig. 1 as a function of $m_{\tilde{u}_R}$ for the case $m_{\tilde{d}_L} = m_{\tilde{d}_R} = 500$ GeV and the same three values of $m_{\tilde{g}}$ considered in the previous figure. The choice $m_{\tilde{d}_L} = m_{\tilde{d}_R}$ is made for simplicity only and does not affect our conclusions in a qualitative way. Except for the special case of highly degenerate right-handed up- and down-squark masses, the $\Delta I = \frac{3}{2}$ gluino box-diagram contribution to ϵ'/ϵ can by far exceed the experimental result,

even taking into account the bounds from Δm_K and ϵ . Indeed, even for moderate splitting the figure implies substantially stronger bounds on $|\text{Im} \delta_{sd}^{LL}|$ than those obtained from $K-\bar{K}$ mixing. This finding is in contrast to the commonly held view that supersymmetric contributions to the electroweak penguin operators have a negligible impact on ϵ'/ϵ . In this context, it is worth noting that a large mass splitting between \tilde{u}_R and \tilde{d}_R can be obtained, e.g., in GUT theories without $\text{SU}(2)_R$ symmetry and with hypercharge embedded in the unified gauge group, without encountering difficulties with naturalness.⁵

The allowed contribution to ϵ'/ϵ increases with the gluino mass (for fixed squark masses) because the $K-\bar{K}$ bounds become weaker in this case. If all supersymmetric masses are rescaled by a common factor ξ , and the bounds from $K-\bar{K}$ mixing are rescaled accordingly, the values for ϵ'/ϵ scale like $1/\xi$. Even for larger squark masses of order 1 TeV the new contribution to ϵ'/ϵ can exceed the experimental value.

In the above discussion we have made no assumption regarding the mass insertion parameter $\text{Im} \delta_{sd}^{RR} \equiv |\delta_{sd}^{RR}| \sin \phi_R$ for right-handed squarks. In models where $|\delta_{sd}^{RR}|$ is not highly suppressed relative to $|\delta_{sd}^{LL}|$, much tighter constraints on $\text{Im} \delta_{sd}^{LL}$ can be derived by applying the severe bounds on the product $\delta_{sd}^{LL} \delta_{sd}^{RR}$ obtained from the chirally-enhanced contributions to $K-\bar{K}$ mixing. Even in this case, however, the supersymmetric contribution to ϵ'/ϵ can still be of order 10^{-3} .³

In summary, we have shown that in supersymmetric extensions of the Standard Model gluino box diagrams can yield a large $\Delta I = \frac{3}{2}$ contribution to ϵ'/ϵ , which only requires moderate mass splitting between the right-handed squarks. In a large region of parameter space, the measured value of ϵ'/ϵ implies a significantly stronger bound on $\text{Im} \delta_{sd}^{LL}$ than is obtained from $K-\bar{K}$ mixing.

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